Towards a Relational Treating of Logic and Language Systems

Lingyuan Ye

October 26, 2020

Tsinghua University

Outline

Philosophical Motivation

Compositionality of Meaning

Two main streams of understanding **meaning** in logic:

- *denotation* (model-theoretic semantics)
- *proof* (proof-theoretic semantics)

In some sense, *compositionality* plays a role in both of the situations.

What Can We Do Without Compositionality?

Key observation:

- The meaning of a sentence in a language is foundamentally involved with its **relations** with the other sentences in the language.

In other words, the meaning of a sentence can be specified without looking at its compositional structures.

A Framework of Relational Semantics

The Flavor of Relational Semantics

Definition (General Logical Frame)

The pair $(\Phi, \rightleftharpoons)$ is called a **general logical frame** if Φ is the set of sentences of some language, and $\rightleftharpoons \subseteq \Phi \times \Phi$ is a binary relation on Φ which is symmetric. In addition, for any $\phi \in \Phi$, either $(\psi, \phi) \notin \rightleftharpoons$ for all $\psi \in \Phi$, such ϕ is called a **contradiction** in Φ, or $(\phi, \phi) \in \rightleftharpoons$.

The Flavor of Relational Semantics

First example:

Definition (Logical Equivalence)

Given a general logical frame $(\Phi, \risingdotseq)$, say two sentenses ψ and ϕ are **logically equivalent**, denoted as $\psi \approx \phi$, if $\psi \risingdotseq \chi$ iff $\phi \risingdotseq \chi$, for all $\chi \in \Phi$.

Intuition: the difference in meaning of two sentences is *witnessed by another sentence*.

The Flavor of Relational Semantics

Second example:

Definition (Logical Consequence)

Suppose Φ is a logical frame. Say ψ is the **logical consequence of** ϕ, denoted as $\phi \vdash \psi$, if $\chi \rightleftharpoons \phi$ implies $\chi \rightleftharpoons \psi$, for all $\chi \in \Phi$.

Normal results:

- EFSQ: for all $\phi \in \Phi$, $\perp \vdash \phi$.
- Dually, for all $\phi \in \Phi$, $\phi \vdash \top$.
- Reflexivity, transitivity, etc.

Further Main Features

- Great generality: a suitable inference reltaion $\vdash \subset \wp(\Phi) \times \Phi$ is built, but also allow the interpretation of many non-classical logic (with further different properties assumed on $\rightleftharpoons$).
- Completeness: A Lindenbaum–Tarski style of algebraic semantics is also built, with respect to which the logic is complete.
- Philosophical ideas: many new things to think about!

The End

Thank you!

Q & A