Craig Interpolation for Subgeometric Logics

Lingyuan Ye, jw. Ivan Di Liberti

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University of Cambridge

Introduction

What is interpolation?

Definition (Craig interpolation syntactically)

Craig interpolation for classical propositional logic says

$$\varphi \vdash \psi \Rightarrow \exists \theta \in \mathcal{L}_\varphi \cap \mathcal{L}_\psi .\ \varphi \vdash \theta \,\&\, \theta \vdash \psi .$$

This is equivalent to pushout squares of Boolean algebras having interpolation.

Definition (Interpolation for posets)

A lax square between posets has *interpolation*, if for any $b \in B$ and $a \in A$, if $vb \leq ua$ then $\exists c \in C.\ b \leq gc \,\&\, fc \leq a$,

$$\begin{tikzcd} C \arrow[r, "f"] \arrow[d, "g"'] \arrow[dr, phantom, "\leq"] & A \arrow[d, "u"] \\ B \arrow[r, "v"'] & D \end{tikzcd}$$

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What is interpolation categorically?

Definition (Interpolation for lex categories)

Consider a lax square between lex categories as shown on the left,

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\begin{tikzcd}
\mathcal{C} \arrow[r, "f"] \arrow[d, "g"'] & \mathcal{A} \arrow[d, "u"] \\
\mathcal{B} \arrow[r, "v"'] \arrow[ur, phantom, "\alpha \nearrow"] & \mathcal{D}
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathrm{Sub}_{\mathcal{C}}(X) \arrow[r, "f"] \arrow[d, "g"'] & \mathrm{Sub}_{\mathcal{A}}(fX) \arrow[d, "\alpha^* u"] \\
\mathrm{Sub}_{\mathcal{B}}(gX) \arrow[r, "v"'] \arrow[ur, phantom, "\leq"] & \mathrm{Sub}_{\mathcal{D}}(vgX)
\end{tikzcd}
$$

It has interpolation if for any $X \in \mathcal{C}$, the lax square of subobjects has interpolation.

Remark

This allows to formulate the syntactic Craig interpolation for multi-sorted first-order theories; (Pitts 1986) has shown all cocomma squares of pretoposes have interpolation.

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Why you may care about interpolation?

- *Category theory*: Interpolation is equivalent to a *Beck-Chevalley* condition: a lax square of posets has interpolation iff their 2-valued presheaves has Beck-Chevalley,

$$\begin{tikzcd} {[D, 2]} \arrow[d, "u^*"'] & {[B, 2]} \arrow[l, "v_!"'] \arrow[d, "g^*"] \\ {[A, 2]} & {[C, 2]} \arrow[l, "f_!"] \end{tikzcd}$$

- *Computer science*: Interpolation has applications in *model checking*, *verification*, *database theory*, *knowledge representation*, etc.
- *Logic*: (Pitts 1986) proves, *constructively*, interpolation for pretoposes implies they are *weakly conceptually complete* w.r.t. any faithful family of universes $\mathcal{V}$:

$$f\colon \mathcal{C} \to \mathcal{D} \vdash f \text{ is an equivalence } \Leftrightarrow \forall V \in \mathcal{V}.\, f^* : \mathbf{Mod}_V(\mathcal{D}) \simeq \mathbf{Mod}_V(\mathcal{C}).$$

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Stories for today

Main plot: To establish Craig interpolation *uniformly* for a general subgeometric logic T.

Three chapters

a) Slicing and propositional bootstrap;
b) An orthogonal factorisation system;
c) A unified proof, with outlook.

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a) Slicing and propositional bootstrap;

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Slicing and propositional bootstrap

Doctrine and interpolation

For now, we represent a fragment by a *doctrine* T, viz. a *lax-idempotent pseudo-monad* T on the 2-category **Lex** of lex categories, with a fully faithful monad map $\sigma \colon \mathsf{T} \Rightarrow \mathbf{Psh}$.

Examples

(κ-)pretoposes, Barr-exact categories, lex categories with strict initial object, lextensive categories, etc. E.g. for pretopos T assigns the free pretopos over a lex category.

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Slicing has a *mapping out* property in **Lex**, by freely adding a global section to an object:

Universal property of slices

For $\mathcal{A} \in \mathbf{Lex}$ and any $x \in \mathcal{A}$, $x^* : \mathcal{A} \to \mathcal{A}/x$ is a *coinserter* in the 2-category **Lex**,[1]

$$\begin{tikzcd} \mathbb{O} \arrow[r, shift left=2, "\mathrm{id}_x"] \arrow[r, shift right=2, "\pi_x"'] \arrow[r, phantom, "\Downarrow\delta_x"] \arrow[d, shift right, "1"'] \arrow[d, shift left, "x"] & \mathcal{A}/x \\ \mathcal{A} \arrow[ur, bend right, "x^*"'] & \end{tikzcd}$$

[1] $\mathbb{O}$ here is the free lex category on 1 object; concretely, $\mathbb{O} \simeq \mathbf{Fin}^{\mathrm{op}}$.

Preserve slicing

Definition (Preserving slicing)

Given a doctrine T with unit η, it *preserves slicing* if for any $\mathcal{A} \in$ **Lex** and any $x \in \mathcal{A}$, the canonical lex functor $(\mathsf{T}\mathcal{A})/\eta x \to \mathsf{T}(\mathcal{A}/x)$ is an equivalence,

Theorem (Di Liberti and Y. 2026)

T *preserves slicing iff* T*-algebras are closed under slicing, with the same universal property,*

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Propositional bootstrap

There is an adjunction between meet-semi-lattices and lex categories, where the right adjoint is a version of propositional truncation (localic reflection),

$$\begin{tikzcd} \mathbf{Lat}_\wedge \arrow[r, bend left] \arrow[r, phantom, "\perp"] & \mathbf{Lex} \arrow[l, bend left, "\mathrm{Sub}_{-}(1)"] \end{tikzcd}$$

Theorem (Di Liberti and Y. 2026)

Let T *preserve slicing. It has the interpolation property iff for any cocomma square of* T*-algebras, the image under the 2-functor* $\mathrm{Sub}_{-}(1)$ *has interpolation.*

Proof.

Universal property of slicing allows us to reduce the interpolation condition for $X \in \mathcal{C}$ as the condition for the terminal object in $\mathcal{C}/X$. □

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A first reflection

- Slicing is important! Can we even imagine a doctrine that does not preserve slicing?
- Incidentally, (Sterling and Y. 2025) shows if a theory $\mathbb{T}$ extending $\mathbf{Lat}_\wedge$ preserves slicing, then in $\mathbf{Set}[\mathbb{T}]$, the generic $\mathbb{T}$-model forms a *dominance* as in (Rosolini 1986).
- We also believe propositional bootstrap is a general phenomenon that applies more generally to other logical properties, and (Di Liberti and Y. 2025) is the starting point for a framework to even formulate such results.

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An orthogonal factorisation system

Filter quotients

Suppose T preserves slicing and is *finitary*, i.e. it preserves filtered pseudo-colimits. By (Di Liberti and Osmond 2022), pretoposes, Barr-exact categories, etc., are all examples.

Definition (Filter quotient)

Let $\mathcal{A}$ be a T-algebra and F a filter on $\mathrm{Sub}_{\mathcal{A}}(1)$. The *filter quotient* is the filtered colimit

$$\mathcal{A} \twoheadrightarrow \mathcal{A}_F \simeq \varinjlim_{\varphi \in F} \mathcal{A}/\varphi.$$

Example

- In logic: Filter quotient corresponds to adding all $\varphi \in F$ as axioms;
- In sheaf-theoretic forcing: Taking a filtered quotient on a ultrafilter constructs a 2-valued Boolean topos from a Boolean topos.
- In HoTT: Filter quotient of model categories later today from Nima Rasekh.

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Truth-conservative maps

Definition (Truth-conservative map)

A lex functor $f\colon \mathcal{A} \to \mathcal{B}$ is *t-conservative* if it reflects truth of subterminal objects

$$\forall \varphi \in \mathrm{Sub}_{\mathcal{A}}(1).\ f\varphi = 1 \Rightarrow \varphi = 1.$$

Remark

Syntactically, this means for any closed formula φ,

$$\vdash_{\mathcal{B}} f\varphi \Rightarrow \vdash_{\mathcal{A}} \varphi.$$

This is weaker than the usual conservativity condition ($f\psi \vdash_{\mathcal{B}} f\varphi$ implies $\psi \vdash_{\mathcal{A}} \varphi$), but the two are equivalent in the presense of function types (implication).

Theorem (Di Liberti and Y. 2026)

Filtered quotients and t-conservative maps form an orth. fact. system on $\mathbb{T}$*-algebras.*

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Let T *be a finitary doctrine that preserves slicing.* T *has interpolation iff in any cocomma square of* T*-algebras below, if* g *is t-conservative, then so is* u*,*

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Proof.

Propositional bootstrap + universal property of filtered quotients. □

Second reflection

- Our orthogonal factorisation system is closely related to (focalisation, terminally connected) orthogonal factorisation system on **Lex** constructed in (Osmond 2021).
- In the tradition of algebraic logic, the interpolation property is often reduced to monos are preserved under pushouts in the corresponding category of algebras for a logic. Our result is a categorification of this.
- The difference between monos and t-conservative maps, and between cocommas and pushouts, is due to the (lack of) existence of *implication*.

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A Unified Proof, with Outlook

A broader picture

To show the exactness property for a wide class of fragments, we work in the framework (Di Liberti and Y. 2025), where a general *classifying topos* construction is available.

Theorem (Di Liberti and Y. 2026)

Any logic (in the sense of loc. cit.) between the regular and coherent fragment that preserves slicing has the interpolation property.

Example

Regular logic with falsum, etc.

Outlook

- Are there any classification of logics in between? Do they always preserve slicing?
- This work belongs to a wider project initiated in (Di Liberti and Y. 2025), where recent progress on *conceptual completeness* (Di Liberti, Tarantino, and Y. 2026).
- More to come: propositional bootstrap, Lindström theorem, etc.

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Reference

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